Does the Crowding-In Effect of Public Spending on Private Consumption Undermine Neoclassical Models?

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Abstract

Empirical evidence from vector autoregressions (VARs) showing that public spending shocks crowd in private consumption has been seen as evidence against standard neoclassical models of the business cycle. We show that a standard real business cycle model in which all agents including the government optimize is compatible with the results from the empirical literature. A VAR estimated using artificial data simulated from the model indicates that, under standard assumptions to identify public spending shocks, an increase in public spending is associated with an increase in private consumption and the real wage. The implied impulse responses are qualitatively and quantitatively similar to those in the empirical literature.

JEL classification: E2, E3, H3

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1. Introduction

Using vector autoregressions (VARs), researchers have found that shocks to public spending crowd in private consumption. Examples include Blanchard and Perotti (2002), Perotti (2007), Fatás and Mihov (2001), and Galí, López-Salido and Vallés (2007).[1] This is cited as a paradox in the context of standard neoclassical models of the business cycle. Without special assumptions (discussed below), both real business cycle (RBC) models and New Keynesian models predict crowding out of private consumption in response to increases in public spending due to a negative wealth effect.[2] When public spending increases, households correctly anticipate an increase in the present value of their taxes. Labour supply increases (which pushes down the equilibrium real wage) and consumption decreases.

We offer a simple explanation for the crowding in result in the empirical literature. We build a standard real business cycle model in which all agents optimize well-defined objective functions subject to technological and budget constraints. Government spending has three components. First, there is public consumption that affects households' utility. Second, public investment increases the stock of public capital, which enters the aggregate production function. Third, there is an exogenous component of spending that affects neither utility nor production. In the model, public consumption and public investment react to exogenous shocks to preferences, technology, and the exogenous component of public spending. Public spending and private consumption respond similarly to the state of the economy since it is optimal for the government to equate the marginal utilities of public and private consumption.[3] Public and private investment also respond sim-

[1] The evidence has been challenged recently in an important paper by Ramey (2011).

[2] See Aiyagari, Christiano and Eichenbaum (1992) and Baxter and King (1993). Barro and King (1984) note that RBC models will in general have trouble explaining positive comovements between output, consumption and investment in response to shocks other than aggregate productivity shocks.

[3] This holds whether private consumption and fiscal spending are substitutes or complements,

ilarly to the state of the economy since it is optimal for the government to equate the marginal product of public capital to that of private capital.

A VAR run on the artificial data and using the standard identifying assumptions from the empirical literature (which are incorrect in the context of our model) leads to the conclusion that innovations to public spending crowd in private consumption. These results obtain even if part of total government spending is exogenous. Furthermore, the impulse response functions of the estimated VAR reproduce qualitatively and quantitatively the response patterns in the empirical literature. This includes a positive comovement between the real wage and government spending.[4] The crowding in result comes from the erroneous assumptions that all of public spending is predetermined with respect to all other variables in the VAR.

A few alternative explanations have been proposed to account for the empirical result of crowding in by public spending shocks.[5] They can be categorized as either being compatible with the neoclassical approach (with optimizing private agents) or not.

In the former category, Bouakez and Rebei (2007) show that the RBC model can generate crowding in when preferences exhibit strong Edgeworth complementarity between public and private spending. Linnemann (2006) obtains the same result with a non-additively separable utility function and a small intertemporal elasticity of substitution. Fève, Matheron and Sahuc (2012) build a neoclassical model in which public spending enters the utility function and one component is endogenous and determined by a countercyclical feedback rule depending on GDP growth. When public spending is an Edgeworth complement to private con-

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and is true to the extent that there are no frictions in goods and labour markets such as nominal rigidities.

[4]Our results are in the spirit of Cooley and Dwyer (1995, page 84) who show in a different context that the identifying restrictions imposed in structural VARs are "uninterpretable without a fully articulated economic model." Their application is to the Blanchard and Quah (1989) analysis of the relative importance of demand and supply shocks.

[5]See Ramey (2011b) for a more detailed survey.

sumption, omitting the endogenous component leads to underestimating the size of the government spending multiplier. Ramey (2011) reconciles the data with the neoclassical approach by overturning the empirical crowding in result. She rules out non-defense spending as a source of independent shocks to public spending, which is in keeping with the neoclassical approach taken here.[6] She models wars as exogenous increases in spending on goods that affect neither utility nor the aggregate production function. When she assumes that these increases in spending were anticipated well in advance, she finds that there is crowding out and not crowding in. Murphy (2015) builds a neoclassical model with imperfect information. Positive government spending shocks increase the incomes of some agents who do not recognize the consequences for future tax liabilities, leading to a positive perceived income effect and the crowding in of consumption.

In the latter category, there are some New Keynesian models that retain optimizing agents except for the decision to change prices and/or wages, which are subject to exogenous rigidities that are not explicitly micro-founded. Zubairy (2014) builds a New Keynesian model with deep habits and variable markups of prices over marginal cost. Markups are countercyclical in her model, allowing wages and consumption to increase in response to a positive public spending shock. Several papers, such as Eggertsson (2010) and Christiano, Eichenbaum and Rebelo (2011), use New Keynesian models to examine the government spending multiplier when nominal interest rates are at their zero lower bound and when there is substantial excess capacity in the economy. Increases in public spending generate a positive income effect and, if expected inflation increases while nominal interest rates are stuck at zero, a fall in the ex ante real interest rate can lead households to substitute intertemporally towards current consumption. Galí, López-Salido and Vallés (2007) set up a New Keynesian model which drops the

[6] However, her defense spending shocks involve the suboptimal appropriation of some of GDP and its destruction.

assumption of optimizing households: a fraction of consumers are constrained to consume their current disposable income in each period, and households are willing to meet the firms' demand for labour at the wage rate set by a union. If the fraction of non-Ricardian consumers is large enough, the model can generate a positive response of consumption to a government spending shock.

The rest of the paper is structured as follows. In the following section, we describe the model and relate it to the existing literature. In the third section, we discuss the model's steady-state properties and its calibration. We present and discuss our results in the fourth section. The fifth section concludes.

2. Model

We model endogenous public spending following Ambler and Cardia (1997). A benevolent government chooses public spending to maximize the welfare of the representative private agent. Kydland and Prescott (1977) showed that optimal government policies are subject to a time inconsistency problem. In our model, the government cannot precommit to its announced policies for public consumption and public investment spending.[7] We use dynamic programming methods to derive time-consistent policies. Private agents and the government have reaction functions that depend on the current state of the economy (Markov strategies). The macroeconomic equilibrium in our model is therefore Markov-perfect. Public spending is partly financed by proportional taxes on labour and capital income.

[7]Most models with endogenous government behavior assume precommitment. Chamley (1986), Chari, Christiano and Kehoe (1991, 1995), and Lansing (1998) used the framework first developed by Ramsey (1927) to consider optimal taxation with precommitment. The existing literature on optimal time-consistent fiscal policies is more sparse. Fischer (1980) compared the levels of welfare that can be attained with and without precommitment in a simple model. Lucas and Stokey (1983) studied how the government can issue nominal debt contracts which make its optimal taxation plans time consistent. Chari and Kehoe (1992) analyzed how trigger strategies can be used as a means of enforcing precommitment. Ortigueira (2006) studied optimal Markov-perfect strategies for financing an exogenous stream of government spending.

Distortionary taxes balance the budget on average. Short-run discrepancies are made up by lump sum taxes. Because of distortionary taxation, the first-best optimum is not attainable.[8]

These assumptions lead to the result that all shocks in the model lead to a positive comovement between private consumption and the endogenous component of public spending for all periods after the shocks hit. They also lead to positive comovements between public and private investment.

2.1 Households

There is a representative private household that values consumption and leisure. Its utility function is given by

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \ln \left(\tilde{c}_{t+i} \right) - \frac{\gamma_t}{1+\psi} n_{t+i}^{1+\psi} \right\}, \tag{1}$$

where E_t is the mathematical expectations operator conditional on information available at time t, β is a subjective discount factor, $\tilde{c}_t$ is the household's total consumption, n_t is the number of hours worked by the household, γ_t is a preference shock, and $\psi \geq 0$ is a preference parameter.

Total consumption is a CES aggregate of private and public consumption expenditures given by

$$\tilde{c}_t = \left(\theta c_t^{-\sigma} + (1-\theta) C_{gt}^{-\sigma} \right)^{-1/\sigma},$$

where c_t is the household's consumption spending, C_{gt} is *per capita* government consumption spending, and the elasticity of substitution between private and public expenditures is $\nu \equiv 1/(1+\sigma)$. The CES specification implies that there are diminishing marginal returns to public spending for a given level of private spending in order to achieve a given level of total consumption. Bouakez and Rebei

[8] See Ambler and Desruelle (1991) for more details on this point.

(2007) showed that the magnitude of the elasticity of substitution ν has crucial implications for the comovement between private and public spending when the latter is determined exogenously. In particular, if the elasticity of substitution is sufficiently low, government spending can crowd in private consumption.

The household has the flow budget constraint given by

$$c_t + i_t \leq (1-\tau_n)\, w_t n_t + (1-\tau_k)\, q_t k_t - T_t, \tag{2}$$

where τ_n and τ_k are respectively the labour and capital income tax rates, w_t is the equilibrium real wage rate, q_t is the equilibrium capital rental rate, and T_t is the *per capita* level of lump-sum taxation.

The household's holdings of capital evolve according to

$$k_{t+1} = (1-\delta)\, k_t + i_t, \tag{3}$$

where δ is the constant rate of depreciation of private capital.

2.2 Firms

The representative firm uses capital and labour services purchased from households to produce goods subject to a production function that has constant returns to scale in private inputs given by

$$Y_t = z_t {N_t}^{\alpha} {K_t}^{1-\alpha} {K_{gt}}^{\alpha_g}, \tag{4}$$

where K_{gt} is the *per capita* stock of public capital at time t, K_t is the *per capita* private capital stock, N_t is the *per capita* number of hours worked,[9] and z_t is an exogenous stochastic process for the state of technology at time t.

[9] We use the convention that when variables appear in both lower and upper case, the lower case variable is a choice or state variable for the individual household while the upper case variable is the equivalent aggregate *per capita* value.

Under perfect competition, factors are be paid their marginal products, so that

$$
\begin{aligned}
w_t &= \alpha z_t \left(K_t/N_t\right)^{1-\alpha} K_{gt}^{\alpha_g}, && (5)\\
q_t &= (1-\alpha)\, z_t \left(N_t/K_t\right)^{\alpha} K_{gt}^{\alpha_g}. && (6)
\end{aligned}
$$

With constant returns to scale in private inputs, factor payments exhaust output, there are no rents, and the α parameter can be calibrated in the standard way from data on labour's share of total income.[10]

2.3 Resource Constraints

The economy's aggregate resource constraint is given by

$$
Y_t \leq C_t + I_t + C_{gt} + I_{gt} + G_{xt}, \tag{7}
$$

and the government's flow budget constraint is given by

$$
C_{gt} + I_{gt} + G_{xt} = \tau_n w_t N_t + \tau_k q_t K_t + T_t, \tag{8}
$$

where I_{gt} is public investment and G_{xt} is an exogenous component of government spending, which affects neither households' utility nor aggregate productivity. The laws of motion for the aggregate private and public stocks of capital are respectively

$$
K_{t+1} = (1-\delta)\, K_t + I_t \tag{9}
$$

and

$$
K_{gt+1} = (1-\delta_g)\, K_{gt} + I_{gt}. \tag{10}
$$

[10]Note that we do not have endogenous growth in our model. The sum of the coefficients on reproducible factors in the production function, $\alpha + \alpha_g$, is less than one in our calibration.

2.4 Shock Processes

Technology, preference, and exogenous government spending shocks evolve according to the stationary AR(1) processes given by

$$\begin{aligned} \ln(z_t) &= (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \epsilon_{zt}, && (11) \\ \ln(\gamma_t) &= (1-\rho_\gamma)\ln(\gamma) + \rho_\gamma \ln(\gamma_{t-1}) + \epsilon_{\gamma t}, && (12) \\ \ln(G_{xt}) &= (1-\rho_x)\ln(G_x) + \rho_x \ln(G_{xt-1}) + \epsilon_{xt}, && (13) \end{aligned}$$

where ρ_z, ρ_γ and ρ_x are strictly bounded between -1 and 1, variables without time subscript denote steady-state values, and ϵ_{zt}, $\epsilon_{\gamma t}$ and ϵxt are normal, uncorrelated white-noise disturbances with standard deviations σ_z, σ_γ and σ_x respectively.

2.5 The Representative Household's Problem

The representative household chooses time paths for $\{n_{t+i}, k_{t+i+1}\}_{i=0}^{\infty}$ in order to maximize the utility function (1). Given the household's choice of employment and its future holdings of capital, its investment expenditures are given by the law of motion for capital, and its private consumption expenditures are given by its flow budget constraint. The household takes as given the wage rate, the rental rate on capital, the government's policy rule, and the feedback rule for the *per capita* equivalents of its choice variables. The household is aware of the government's flow budget constraint, and is able to calculate the level of lump sum taxes necessary to balance its budget.

This problem can be expressed as a stationary discounted dynamic programming problem. The one-period return function of the household can be written as

$$r_t^h(Z_t, G_t, S_t, s_t, D_t, d_t) = \ln(\tilde{c}_t) - \frac{\gamma_t}{1+\psi} n_t^{1+\psi}, \tag{14}$$

where $\tilde{c}_t$ is given by equation (1). The household's budget constraint is

$$c_t = (1-\tau_{nt}) w_t n_t + (1-\tau_{kt}) q_t k_t - T_t - k_{t+1} + (1-\delta) k_t,$$

where lump sum taxes are given by the government's flow budget constraint, and where

$$Z_t = \{z_t, \gamma_t, G_{xt}\}$$

is a vector of state variables which are exogenous from the point of view of the representative household,

$$G_t = \{K_{gt+1}, C_{gt}\}$$

is a vector of government control variables whose laws of motion are also exogenous from the point of view of the household,

$$S_t = \{K_{gt}, K_t\}$$

is a vector of the *per capita* equivalents of the household's state variables,

$$s_t = \{K_{gt}, k_t\}$$

is a vector of the household's state variables themselves,[11]

$$D_t = \{N_t, K_{t+1}\}$$

is the vector of *per capita* equivalents of the household's control variables, and

$$d_t = \{n_t, k_{t+1}\}$$

are the control variables themselves. The household's value function can be written as

$$v^h(Z, G, S, s) =$$

$$\max_d \left\{ r^h(Z, G, S, s, D, d) + \beta \mathrm{E}\left[v^h(Z', G', S', s') \mid Z, G \right] \right\}, \tag{15}$$

[11]Even though the representative household cannot control the evolution of K_{gt}, the numerical solution method we use makes it convenient to include K_{gt} as an element of its state vector.

where we have dropped time subscripts, where primes denote next-period values, and where

$$Z' = A(Z) + \epsilon',$$

$$s' = B(Z, G, S, s, D, d),$$

$$S' = B(Z, G, S, S, D, D),$$

$$G = G(Z, S),$$

$$D = D(Z, G, S).$$

The household takes as exogenous the government's feedback rule given by $G = G(Z, S)$. In equilibrium, this feedback rule must also satisfy the government's optimality conditions. The solution to the household's dynamic programming problem gives a feedback rule of the form

$$d = d(Z, G, S, s). \tag{16}$$

2.6 Maximization by the Government

The government chooses time paths for $\{C_{gt+i}, K_{gt+i+1}\}_{i=0}^{\infty}$ to maximize the utility of the representative household. Public investment is then given by the law of motion for the public capital stock, and T_t is determined in order to satisfy the government's flow budget constraint. Because taxes are distortionary, the government cannot attain a first-best optimum.[12] The government takes as given the economy's resource constraint and the laws of motion for the aggregate capital stocks. It knows the private sector reaction function given by (16), and takes into account the effects of its actions on the private sector. Because of this, it acts as

[12] See Chari, Kehoe and Prescott (1989) or Ambler and Desruelle (1981).

a Stackelberg leader with respect to the private sector.[13] We use dynamic programming techniques to derive its optimal strategy, so the government's policies are time-consistent by construction. The government's one-period return function can be written as

$$r^g\left(Z_t, S_t, D_t, G_t\right) = \ln\left(\tilde{C}_t\right) - \frac{\gamma_t}{1+\psi} N_t^{1+\psi}, \tag{17}$$

with

$$\begin{aligned}
\tilde{C}_t &= \left(\theta C_t^{-\sigma} + (1-\theta)\, C_{gt}^{-\sigma}\right)^{-1/\sigma}, \\
C_t &= Y_t - I_t - I_{gt} - C_{gt} - G_{xt}.
\end{aligned}$$

Given this return function, the government's value function can be written as

$$v^g\left(Z, S\right) = \max_{G} \left\{ r^g\left(Z, S, D, G\right) + \beta \mathrm{E}\left[v^g\left(Z', S'\right) \mid Z\right] \right\}. \tag{18}$$

The solution to the government's problem gives a feedback rule of the form $G = G\left(Z, S\right)$, which is taken by households as a constraint in their maximization problem.

2.7 General Equilibrium

The following conditions must hold in general equilibrium.

- All agents maximize given their constraints.

- The optimal feedback rule for the representative household is compatible with the feedback rule for the *per capita* counterparts of its choice variables, so that

$$d\left(Z, G, S, S\right) = D\left(Z, G, S\right).$$

[13]Both the government and the household observe current-period shocks before making their decisions. Therefore, shocks to technology, preferences, and public spending shocks will affect the household's and the government's control variables.

- The law of motion for the government's control variables that is a constraint in the representative household's dynamic programming problem is compatible with the optimal feedback rule that is the solution to the government's problem.

- Markets clear.

All agents solve dynamic programming problems. Their policy functions depend on the current state of the economy. General equilibrium in the model can therefore be characterized as Markov-perfect.

3. Steady State and Calibration

The optimization problems of the household and the government cannot be solved analytically. We used numerical techniques (described in more detail in Ambler and Paquet, 1994) that are an extension of those discussed in Hansen and Prescott (1995).[14] We used the household's and the government's exact first order conditions to calculate the deterministic steady state of the model (the long run equilibrium the economy would reach in the absence of stochastic shocks), and then calculated quadratic approximations of the one-period return functions of the household and government around this point. This gives linear feedback rules for the household and government and quadratic value functions, and simple iterative techniques give the optimal feedback rules and value functions.[15] The steady-state properties of the model were used to calibrate some of its parameters. The model was calibrated to U.S. quarterly data.

[14] Ambler and Pelgrin (2010) describes an extension that preserves nonlinearities in the solution.

[15] First-order approximations are adequate for analyzing the model's time series properties. As shown by Kim and Kim (2003), we would need second-order approximations for valid welfare comparisons of different policies.

The parameter values used in our base-case simulations are summarized in Table 1. The subjective discount rate, β, the depreciation rates δ and δ_g, and the share parameter α were set to standard values from the real business cycle literature. The tax rates τ_n and τ_k were set to 0.197 and 0.313, respectively.

The first order conditions for the representative household were then used to calibrate the parameters of the utility function. The first order conditions for the representative household with respect to its control variables are

$$\frac{\partial v^h}{\partial d} = \frac{\partial r^h}{\partial d} + \beta \frac{\partial v^h}{\partial s'} \frac{\partial s'}{\partial d} = 0.$$

Differentiating the value function with respect to the current states s and making use of the first order conditions gives

$$\frac{\partial v^h}{\partial s} = \frac{\partial r^h}{\partial s} + \beta \frac{\partial v^h}{\partial s'} \frac{\partial s'}{\partial s}.$$

In the steady state, this gives

$$\frac{\partial v^h}{\partial s} = \frac{\partial r^h}{\partial s} \left(\underline{I} - \beta \frac{\partial s'}{\partial s} \right)^{-1},$$

where $\underline{I}$ is the identity matrix, so that the first order conditions for the household in the steady state become

$$\frac{\partial r}{\partial d} + \beta \frac{\partial r^h}{\partial s} \left(\underline{I} - \beta \frac{\partial s'}{\partial s} \right)^{-1} \frac{\partial s'}{\partial d} = 0. \tag{19}$$

Applying this equation to our model and imposing the aggregate consistency conditions gives the following equations:

$$\begin{aligned} \frac{1}{C} (1 - \tau_n) w - \gamma N^{\psi} &= 0, && (20) \\ \beta \{ (1 - \tau_k) q + (1 - \delta) \} - 1 &= 0. && (21) \end{aligned}$$

The last equation gives a solution for the rental rate of capital in the steady state that depends only on the discount rate, the depreciation rate of capital, and the rate of taxation on capital income. Given this solution for q, it is possible to solve for the equilibrium steady-state private capital stock using equation (6), for given values of N and K_g. Then, for a given level of hours, we can back out the value of γ consistent with this equilibrium. We calibrated the model so that the average number of hours per employee N matched its average *per capita* value in the U.S. data. We chose a low value for ψ, which increases the variability of total employment.

The first order conditions for the government can be written as

$$\frac{\partial v^g}{\partial G} = \frac{\partial r^g}{\partial G} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial G} + \beta \frac{\partial v^g}{\partial S'}\left(\frac{\partial S'}{\partial G} + \frac{\partial S'}{\partial D}\frac{\partial D}{\partial G}\right) = 0,$$

where $\partial D/\partial G$ gives the effects of a change in the government's control variables on the behavior of the private sector. Differentiating the government's value function with respect to the current states S and using the first order conditions gives

$$\frac{\partial v^g}{\partial S} = \frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S} + \beta\left(\frac{\partial v^g}{\partial S'}\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial v^g}{\partial S'}\frac{\partial S'}{\partial S}\right).$$

At the steady state, this gives

$$\frac{\partial v^g}{\partial S} = \left(\frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S}\right)\left(\underline{I} - \beta\left(\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial S'}{\partial S}\right)\right)^{-1}.$$

Evaluating the first order conditions at the steady state and substituting this expression for the partial derivatives of the value function with respect to the states gives:

$$\begin{gathered}\frac{\partial r^g}{\partial G} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial G} \\ +\beta\left(\frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S}\right)\left(\underline{I} - \beta\left(\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial S'}{\partial S}\right)\right)^{-1}\cdot \\ \left(\frac{\partial S'}{\partial G} + \frac{\partial S'}{\partial D}\frac{\partial D}{\partial G}\right) = 0. \end{gathered} \tag{22}$$

This gives two equations to solve for the steady-state levels of the government's control variables C_{gt} and K_{gt+1}, given the solutions for the steady-state levels of the household's control variables. Alternatively, the steady-state levels of C_g and K_g can be imposed, and the first-order conditions can be used to back out values of θ and α_g compatible with these levels.

These equations are complicated to solve. First, it is necessary to evaluate the partial derivatives of the private control variables with respect to the model's state variables and with respect to the government's controls. This involves either taking total differentials of the household's first order conditions evaluated at the steady state, or solving for the household's optimal feedback rule, which necessitates having solved for the steady state of the model.

In order to circumvent these difficulties, we proceeded as follows. We chose values for C_g and I_g to match the average ratios of current government consumption to output and of public investment to output from our data set. Then, for given values of the θ and α_g parameters and given steady-state values of C_g and I_g, as well as a given feedback rule for the government, we solved the model numerically for the private sector's optimal feedback rule. We then took this private feedback rule as given and derived the optimal feedback rule for the government, which gave implied steady-state values for C_g and I_g. For any discrepancy between the initial and implied values of the government controls, the θ and α_g parameters were adjusted in value, and we iterated until we arrived at values for θ and α_g consistent with the initial postulated steady-state equilibrium, and until the household's and government's value functions converged.[16]

The parameters of the stochastic process for z_t were calibrated to standard values from the RBC literature; the value of z is an arbitrary normalization. The parameters for the preference shock are taken from Chang, Gomes and Schorfheide

[16]Klein, Krusell and Ríos-Rull (2008) solve for the steady state of a similar model by using only steady-state information. They approximate the decision rules by taking successively higher-order polynomial approximations and truncating the polynomials when the steady state changes by less than some convergence criterion.

(2002); as noted above the constant γ is chosen so that the steady-state value of hours as a fraction of the time endowment matches the average in the data. The parameters of the process for G_{xt} are based on the estimates in Bouakez, Chihi and Normandin (2014).

Finally, the elasticity of substitution parameter σ was set to -0.5 so that private and public consumption are substitutes. The steady-state properties of the model are summarized in Table 2 below. The steady-state level of average hours and the ratios of the components of different aggregates to GNP reproduce their sample averages in U.S. data.

4. Results

We first computed the theoretical impulse responses of variables to an innovation to the G_{xt} process that increases exogenous spending by one percent. Figure 1 below shows the responses of C_t, C_{gt}, I_{gt}, and total public spending. Total public spending increases, but both public consumption and public investment decrease in response to the shock. An exogenous increase in government spending crowds out private consumption. This crowding out result is the basis for the argument that the empirical evidence undermines neoclassical models.

We then used the model to simulate 1000 sequences of artificial series for output, public spending, private consumption, private investment, the real rental rate, and the real wage. Each series has a length of 300 periods. In each iteration, the first 100 observations were discarded to ensure that the results did not depend on initial conditions. The number of remaining observations roughly corresponds to the sample size used in empirical studies based on quarterly data.

Using the simulated series, we estimated a 4th-order VAR similar to those found in the empirical literature.[17] Like Fatás and Mihov (2001), Galí, Lopez-

[17]Because the model has only three shocks, stochastic singularity prevents us from estimating

Salido and Vallés (2007) and Bouakez and Rebei (2007), we identified government spending shocks by imposing a causal ordering on the contemporaneous shocks using a diagonalization of the variance-covariance matrix of the residuals. More specifically, our identification scheme implies that innovations to government spending affect all the remaining variables contemporaneously, whereas innovations to these variables affect government spending only with a lag.[18]

In each iteration, we used these restrictions to compute the impulse response functions to a 1 percent government spending innovation. The responses, represented with solid lines in Figures 2 and 3, are averages across the 1000 replications. Their confidence intervals, delimited with dotted lines, were computed by excluding the 2.5 percent lowest and highest responses. Figure 2 depicts the case where private and public consumption are substitutes.

It shows that an orthogonalized positive innovation to public spending generates a large and persistent increase in private consumption. Interestingly, the consumption response has a hump-shaped pattern, reaching its peak around 12 quarters after the shock, which accords with much of the evidence reported in the empirical literature. The response of the real wage is also positive at all horizons as well as hump shaped, as documented in many earlier empirical studies. Figure 1 shows that an increase in the component of government spending that is truly exogenous leads to crowding out. In the VAR government spending is not decomposed into its truly exogenous and endogenous components. Rather, government spending shocks are (mistakenly) identified by the assumption of causal ordering of the error terms. The VAR is picking up the positive comovement between pri-

a VAR with the six simulated series at once. To circumvent this problem, we estimated four different 3-variable VARs that have in common government spending and output but where the third variable is either private consumption, private investment, the real rental rate or the real wage. We varied the lag length from 1 to 8 and found the results to be extremely robust.

[18] Blanchard and Perotti (2002) do not use a purely recursive identification strategy, but they assume that government expenditures are predetermined relative to output and taxes.

vate and public consumption that comes from the equating their marginal utilities.

As a robustness exercise, we redid the simulation exercise with $\sigma = 2.0$, the case where private and public consumption are complements. We obtained very similar results, both qualitatively and quantitatively, when private and public expenditures were assumed to be complements. The responses are in Figure 3 below. In particular, there is a large, persistent and non-monotonic crowding-in effect on consumption. In addition, the response of the real wage is positive at all horizons and is hump shaped, and the responses of the other variables are very similar.

To summarize, when some of public spending is set optimally, a VAR estimated using the simulated series, and which identifies innovations to government spending as is commonly done in the literature, leads to the conclusion that public spending shocks crowd in private consumption, regardless of whether private and public expenditures are substitutes or complements. This is despite the fact that the data generating process does not depart from the standard real business cycle model, except for the way the government makes its decisions. Therefore, the conclusion that RBC models are inconsistent with the data is unwarranted.

In order to gain some intuition about the mechanism that allows the model with optimal public spending to generate a crowding-in effect, it is instructive to examine the theoretical response of private and public spending to the different (true) structural shocks in the model economy. Responses to technology and preference shocks are depicted in Figures 4 and 5 respectively. Figure 4 shows that private consumption and the two components of public spending (i.e., public consumption and investment) increase in response to a technology shock. Private and public consumption are responding optimally to the positive wealth effect of the technology shock. Public investment responds optimally to the persistent increase in the marginal productivity of public capital. On the other hand, a preference shock leads to a fall in private and public spending. Private and public consumption optimally fall as private agents place more weight on leisure. Private and public investment optimally fall as the persistent decrease in hours lowers the marginal productivity of private and public capital. In sum, private and public spending tend to move together in response to each of the structural shocks.

5. Conclusion

We simulated a model in which public consumption and investment spending are determined by a government that maximizes a well-defined objective function. The model generates positive comovements between public spending and private consumption that are compatible with recent evidence from vector autoregressions. When artificial data from simulations of the model are used to estimate vector autoregressions and when the same assumptions as in the empirical literature are used to identify government spending shocks, these shocks appear to crowd in private consumption.

Furthermore, the impulse response functions from the estimated VARs are broadly compatible with the VAR evidence. The responses of consumption and other variables to measured public spending shocks are hump shaped. Measured positive shocks to public spending lead to increases in the real wage, which also conforms to the evidence from VARs.

Our model offers a simple explanation that can reconcile standard neoclassical theory with the empirical evidence. The model we develop is in keeping with the principle of treating all agents as optimizing well-defined objective functions subject to technological and budget constraints.

In our simulations, some of government spending is endogenous. We would argue that it is extremely difficult to identify truly exogenous components of public spending in the data. A common strategy in the empirical literature is to equate the exogenous component of public spending with military spending. In this approach, shocks to military spending are modelled as changes in the quantity of aggregate output that is confiscated by the government and destroyed, with no benefit to households' utility or aggregate production. Changes in military spending may very well be optimal responses to threats to external security such as attacks or threats of attacks from foreign powers, making even military spending endogenous. Such shocks would also have repercussions on labour supply, labour demand, and the marginal utility of private consumption, necessitating a richer approach to modelling wars.

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Table 1: Parameter Calibration

Parameter	Value
α	0.640
α_g	0.050
δ	0.021
δ_g	0.021
β	0.990
ψ	0.050
θ	0.722
σ	-0.500
z	1.000
ρ_z	0.950
σ_z	0.007
γ	0.687
ρ_γ	0.940
σ_γ	0.0089
G_x	0.050
ρ_x	0.8
σ_x	0.008
τ_n	0.197
τ_k	0.313

Table 2: Steady State

Variable	Value
N	0.352
Y	1.155
C/Y	0.679
I/Y	0.167
C_g/Y	0.087
I_g/Y	0.026
G_x/Y	0.043
K/Y	7.951
K_g/Y	1.137

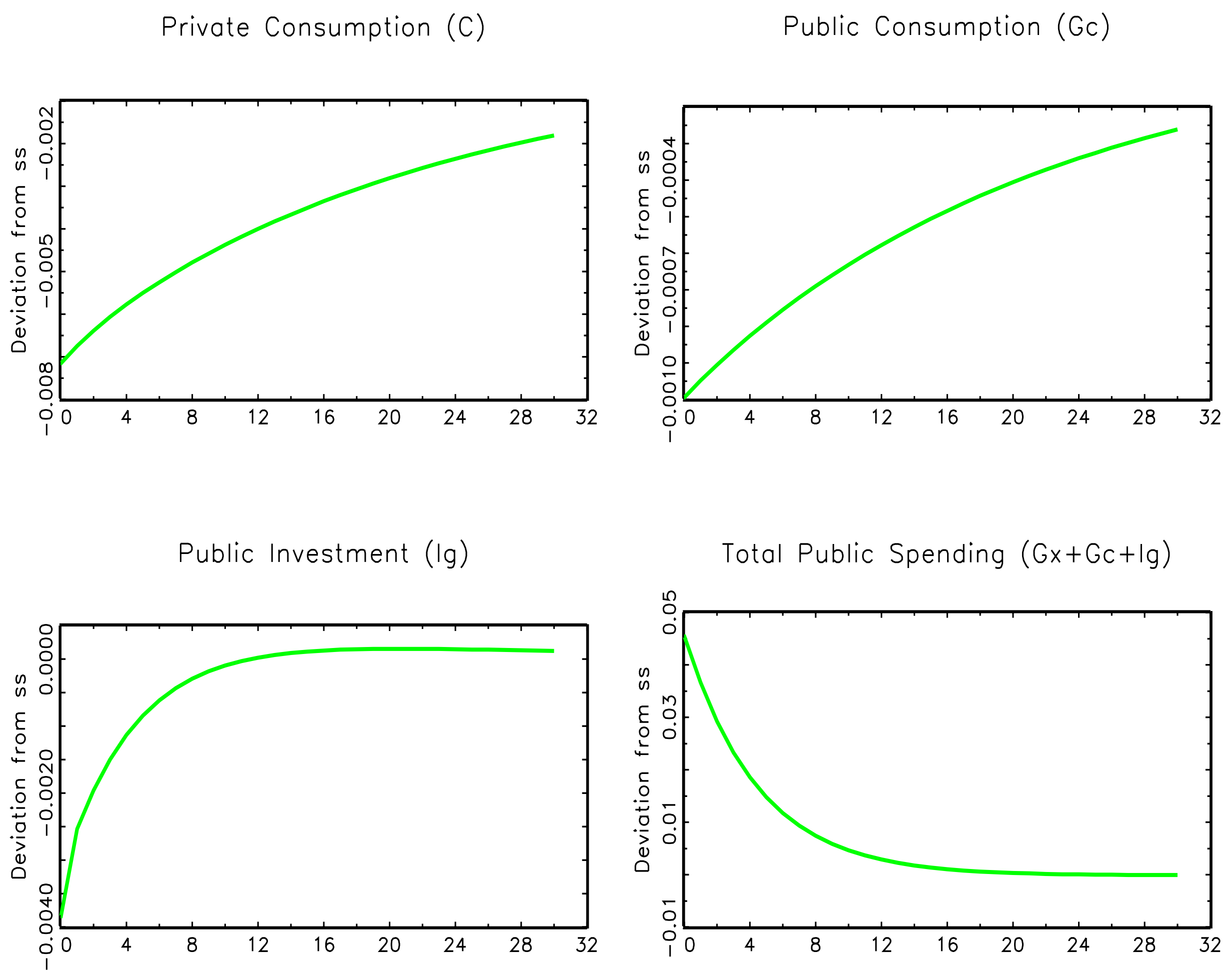

Figure 1: Theoretical impulse responses to a one percent increase in the exogenous component of public spending

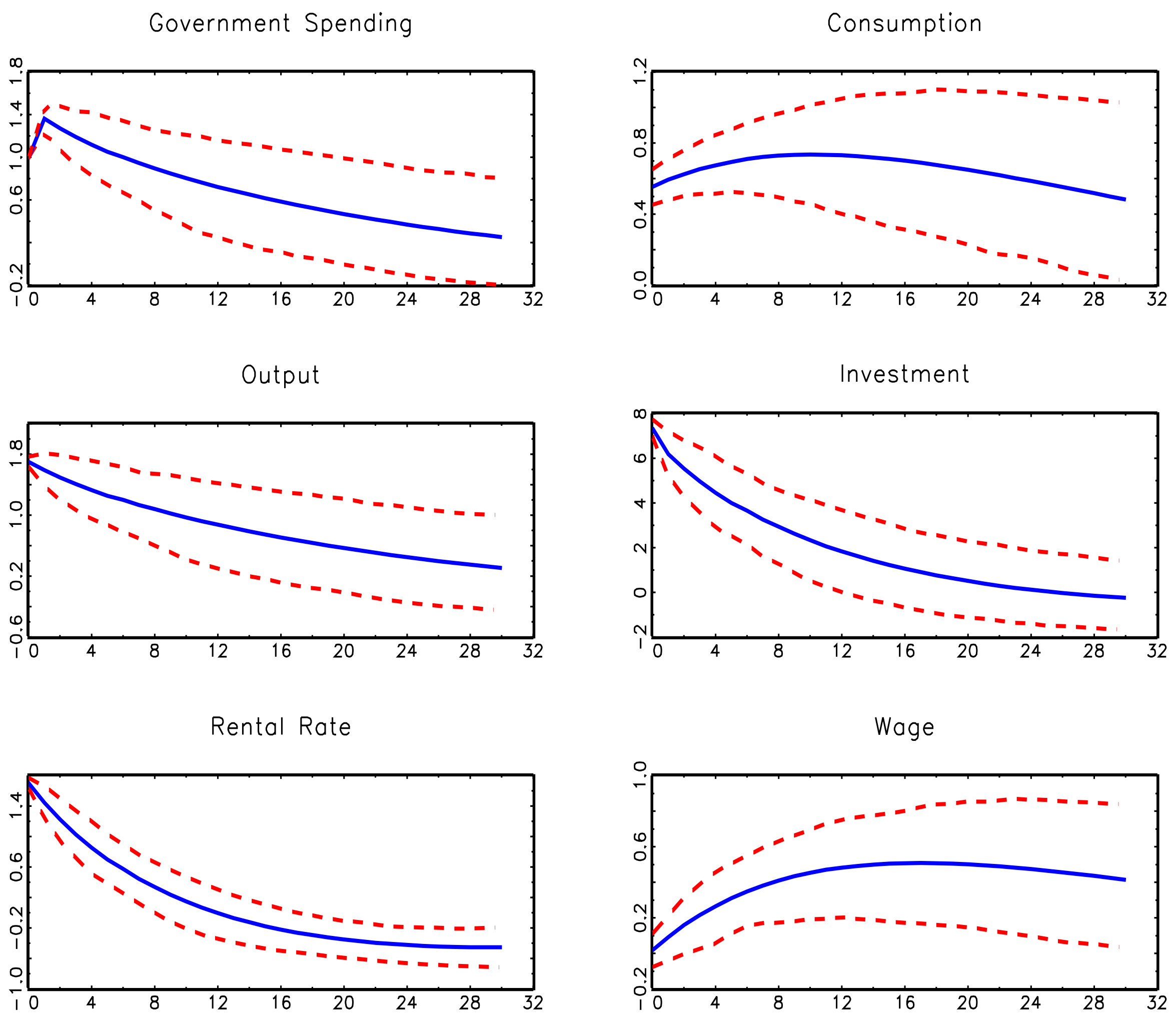

Figure 2: Estimated impulse responses to a 1 per cent increase in public spending (private and public consumption are substitutes)

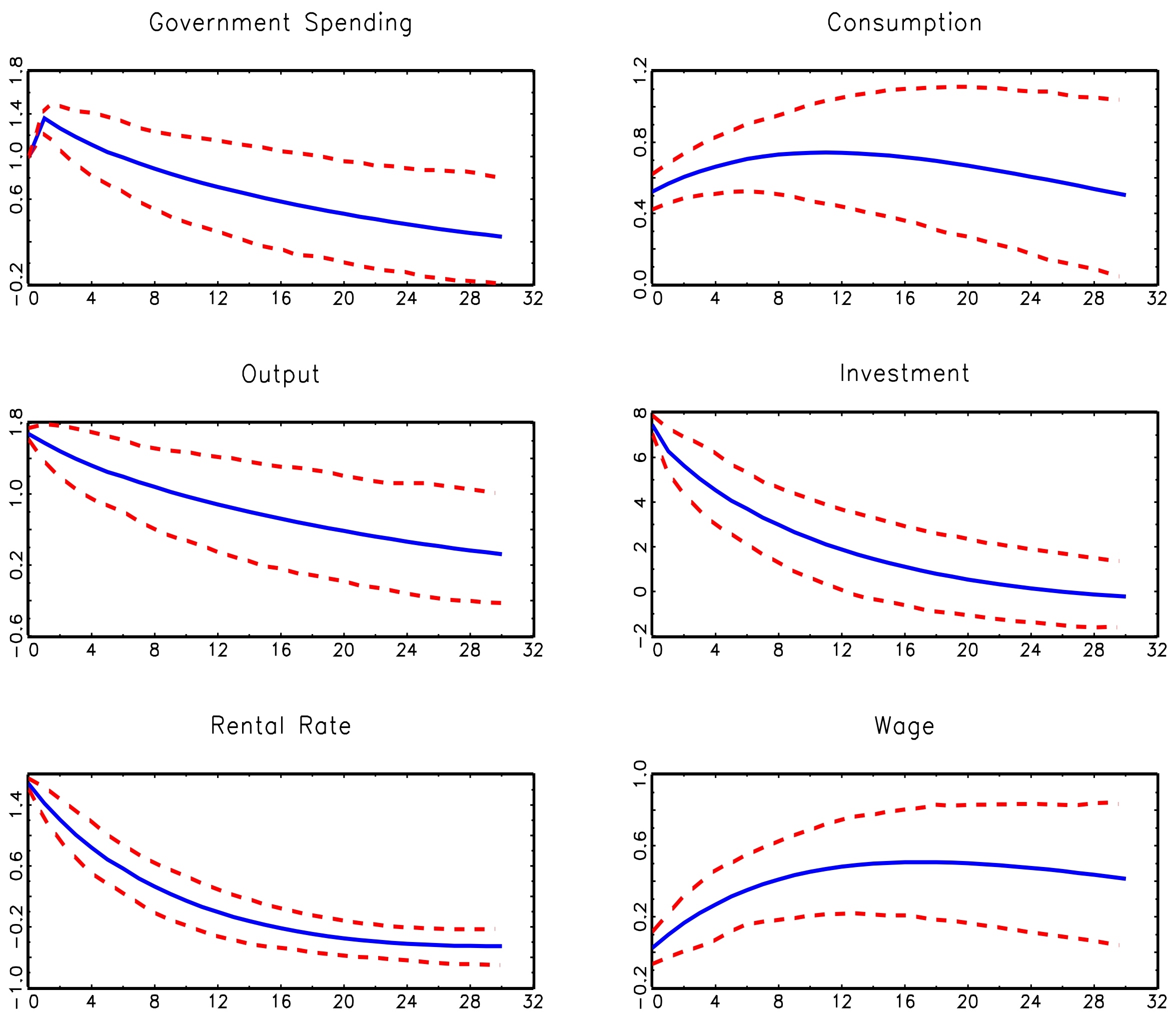

Figure 3: Estimated impulse responses to a 1 per cent increase in public spending (private and public consumption are complements)

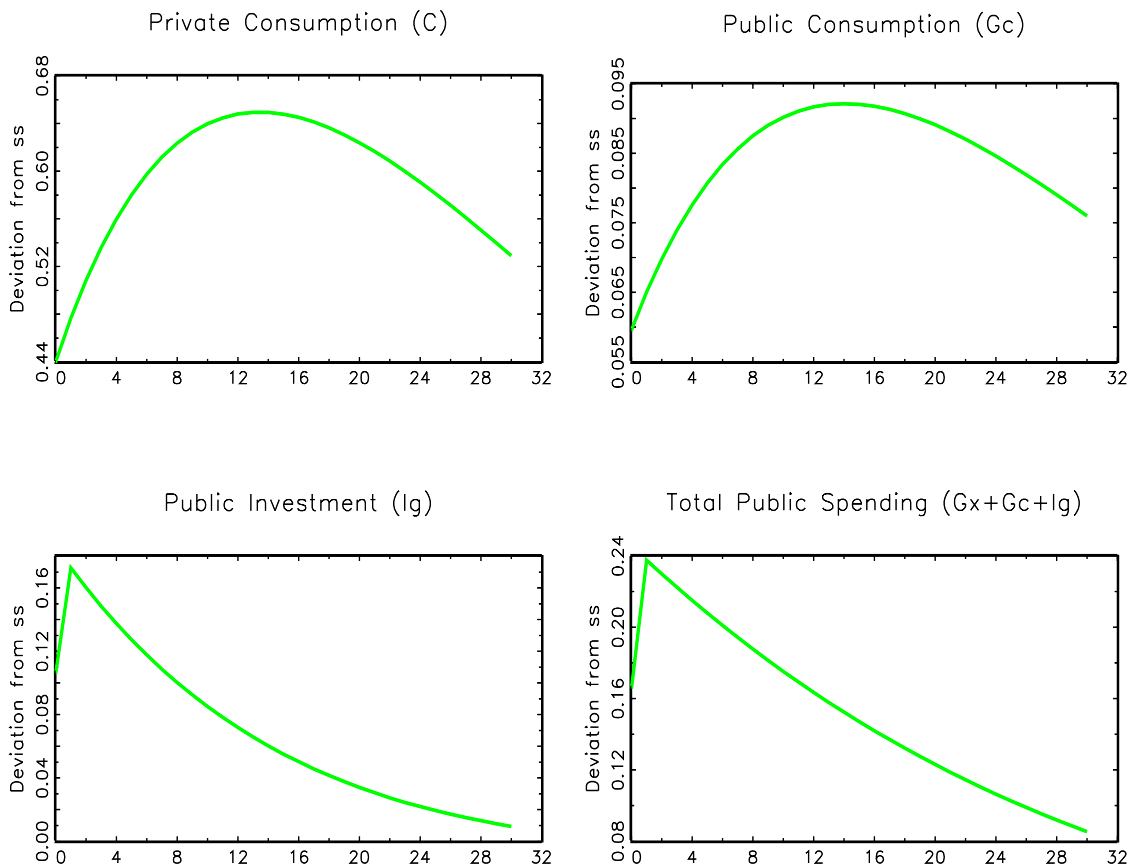

Figure 4: Theoretical impulse responses to a 1 per cent technology shock.

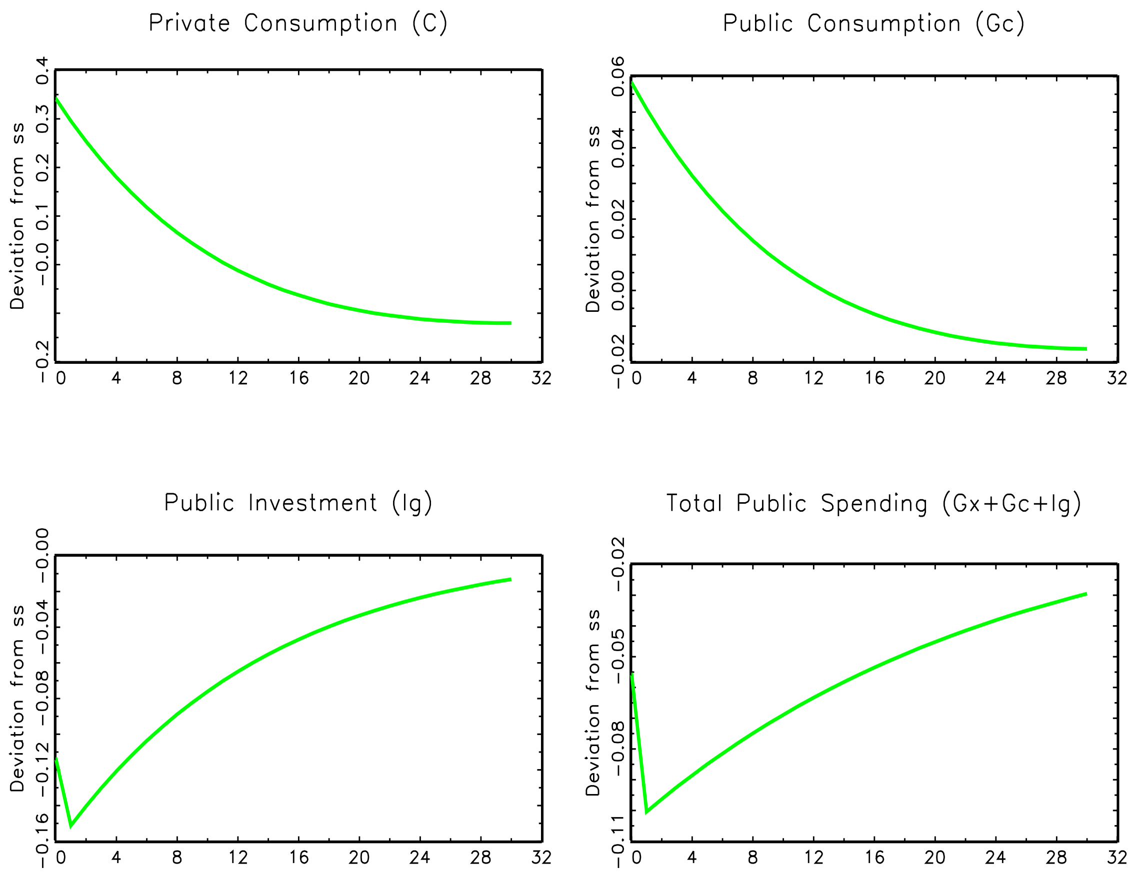

Figure 5: Theoretical impulse responses to a 1 per cent preference shock.